A new Age for Turbulence: DNS, LES, URANS ...

ONERA Châtillon, France, 02 December 2011

The new partially integrated transport modeling (PITM) method for continuous hybrid non-zonal RANS/LES simulations of turbulent flows

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OUTLINE

- From RANS to LES modeling
- Partial Integrated Transport Modeling (PITM) method: Hybrid RANS/LES simulations
  - Mathematical physics formalism developed in the spectral space
  - Transport equation for the subfilter scale stress
  - Transport equation for the subfilter dissipation rate
- Engineering applications
  - Injection induced flows (space launchers)
  - Channel flow with streamwise constrictions (aeronautics industry)
  - Channel flows subjected to spanwise rotation (turbomachinery)

FROM RANS TO LES MODELING

- RANS modeling : many contributions in the past forty years
  - First and second order closures (Launder, Lumley, Speziale, Gatski, Rodi et al...)
- Academic large eddy simulation
  - Smagorinsky (1963), dynamic Smagorinsky (Piomelli and Germano, 1991)
  - Structure-function model (Lesieur et al. , 1996) etc ....
- Hybrid zonal approach
  - Detached-Eddy simulation DES (Spalart et al., 2000)
- Hybrid continuous approach
  - PITM method (Schiestel, Chaouat, Dejoan 2005-2011)
  - TPITM method (Manceau, Gatski, Fadai-Ghotbi et al., 2007-2011)
  - Scale-adaptative simulation SAS (Menter et al., 2005-2011)
  - PANS method (Girimaji et al., 2006-2011)

TURBULENCE MODELING

- **Transport equation for the statistical velocity** $\langle u_i \rangle$**RANS approach**

$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial}{\partial x_j}\Big(\langle u_i \rangle \langle u_j \rangle \Big) = -\frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i \rangle}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \quad \textbf{(1)}$$

with $\tau_{ij} = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle$

- **Transport equation for the filtered velocity** $\bar{u}_i$ **LES and continuous HYBRID approaches**

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial (\tau_{ij})_{sfs}}{\partial x_j} \quad \textbf{(2)}$$

with $(\tau_{ij})_{sfs} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$

- **Second order closure is based on the transport equation of the tensor** τ_{ij} **or** $(\tau_{ij})_{sfs}$

PARTIALLY INTEGRATED TRANSPORT MODELING (PITM) METHOD

- **Objective: to perform large eddy simulations of turbulent flows on relatively coarse grids**
- **Bridge between URANS and LES method with seamless coupling**
- **Self consistency of the PITM method obtained when the cutoff location continuously varied between two extreme limits (DNS/PITM/RANS)**

$$\lim_{\kappa_c \to 0} [(\tau_{ij})_{sfs}] = (\tau_{ij})_{RANS} \quad (3)$$

$$\lim_{\kappa_c \to \infty} [(\tau_{ij})_{sfs}] = 0 \quad (4)$$

- **Definition of the subfilter-scale tensor** $(\tau_{ij})_{sfs} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$
- **Definition of the resolved scale tensor** $(\tau_{ij})_{les} = \bar{u}_i \bar{u}_j - \langle u_i \rangle \langle u_j \rangle$ **where** $\langle . \rangle$ **denotes the statistical average**
- **Definition of the Reynolds stress tensor** τ_{ij} **including the small and large scale fluctuating velocities** $\tau_{ij} = \langle (\tau_{ij})_{sfs} \rangle + \langle (\tau_{ij})_{les} \rangle$

MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- Cooperation between ONERA (Chaouat) and CNRS/IRPHE (Schiestel)
  - Spectral partitioning (m = number of zones), definition of filtered and averaged quantities

$$u_i = \langle u_i \rangle + \sum_{m=1}^{N} u_i^{'(m)} \; ; \; u_i^{'(m)}(\boldsymbol{\xi}) = \int_{\kappa_{m-1}<|\kappa|<\kappa_m} \widehat{u'}_i(\boldsymbol{\kappa}) \exp\left(j\boldsymbol{\kappa}\boldsymbol{\xi}\right) d\boldsymbol{\kappa} \qquad \textbf{(5)}$$

  - Simulation LES (m=2) : filtered velocity: $\bar{u}_i = \langle u_i \rangle + u_i^{'(1)}$
    * Large-scale fluctuating velocity: $u_i^{<} = u_i^{'(1)}$
    * subgrid-scale fluctuating velocity: $u_i^{>} = u_i^{'(2)}$

MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Two-point velocity fluctuating correlation for non-homogeneous turbulence**
  $R_{ij} = \langle u'_{iA} u'_{jB} \rangle (\boldsymbol{x}_A, \boldsymbol{x}_B)$ **(Hinze, 1975)**
- **New independent variables**
  - **vector difference** $\boldsymbol{\xi} = \boldsymbol{x}_B - \boldsymbol{x}_A$
  - **midway position** $\boldsymbol{X} = \frac{1}{2}(\boldsymbol{x}_A + \boldsymbol{x}_B)$
- **Transport equation for the tensor** $R_{ij} = \langle u'_{iA} u'_{jB} \rangle (\boldsymbol{X}, \boldsymbol{\xi})$
- **Taylor series development for the mean velocity (framework of tangent homogeneous spectral space, Schiestel, 1987; Chaouat and Schiestel, 2007)**
- **Fourier transform of the transport equation for the tensor** $\widehat{R_{ij}}(\boldsymbol{\kappa}, \boldsymbol{X})$
- **Integration on a spherical shell in the wave numbers (Schiestel, 1987; Cambon et al., 1992; Chaouat and Schiestel, 2007)**

$$\varphi_{ij}(\kappa, \boldsymbol{X}) = (R_{ij}(\boldsymbol{X}, \boldsymbol{\xi}))^{\Delta} = \frac{1}{A(\kappa)} \iint_{\partial A} \widehat{R_{ij}}(\boldsymbol{\kappa}, \boldsymbol{X}) dA(\boldsymbol{\kappa}) \qquad \textbf{(6)}$$

- **Partial integrations on the wavenumbers to return in the physical space**

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MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Resulting equation in the spectral space by mean integrations over spherical shells, $\varphi_{ij}(\boldsymbol{X},\kappa) = (R_{ij}(\boldsymbol{X},\boldsymbol{\xi}))^{\Delta}$ (Chaouat and Schiestel, 2007)**

$$\frac{D\varphi_{ij}(\boldsymbol{X},\kappa)}{Dt} = \mathcal{P}_{ij}(\boldsymbol{X},\kappa) + \mathcal{T}_{ij}(\boldsymbol{X},\kappa) + \Pi_{ij}(\boldsymbol{X},\kappa) + \mathcal{J}_{ij}(\boldsymbol{X},\kappa) - \mathcal{E}_{ij}(\boldsymbol{X},\kappa) \qquad (7)$$

- **Production term $\mathcal{P}_{ij}$, Transfer term $\mathcal{T}_{ij}$, Redistribution term Π_{ij}, Diffusion term $\mathcal{J}_{ij}$, Dissipation term $\mathcal{E}_{ij}$**

MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Integration in the spectral space for recovering physical space quantities**
  - **Statistical one-scale models in the physical space**

$$R_{ij} = \int_0^\infty \varphi_{ij}(\boldsymbol{X},\kappa)d\kappa \quad => \quad \frac{DR_{ij}}{Dt} = P_{ij} + \Phi_{ij} + J_{ij} - \epsilon_{ij} \tag{8}$$

  - **Statistical multiple-scale models in the physical space, (Schiestel, 1987)**

$$R_{ij}^{(m)} = \int_{\kappa_{m-1}}^{\kappa_m} \varphi_{ij}(\boldsymbol{X},\kappa)d\kappa \quad => \quad \frac{DR_{ij}^{(m)}}{Dt} = P_{ij}^{(m)} + F_{ij}^{(m-1)} - F_{ij}^{(m)} + \Phi_{ij}^{(m)} + J_{ij}^{(m)} - \epsilon_{ij}^{(m)} \tag{9}$$

  - **Subfilter-scale models in the physical space, (Chaouat and Schiestel, 2005)**

$$\langle(\tau_{ij})_{sfs}\rangle = \langle u_i^> u_j^> \rangle = R_{ij}^{(2)} \quad => \quad \frac{D\langle(\tau_{ij})_{sfs}\rangle}{Dt} = P_{ij}^{(2)} + F_{ij}^{(1)} - F_{ij}^{(2)} + \Phi_{ij}^{(2)} + J_{ij}^{(2)} \tag{10}$$

by analogy :

$$\frac{D(\tau_{ij})_{sfs}}{Dt} = ... \tag{11}$$

MATHEMATICAL PHYSICS FORMALISM IN PHYSICAL SPACE

- **Resulting equation in the physical space, (Chaouat and Schiestel, 2007)**

$$\frac{DR_{ij}^{(m)}}{Dt} = P_{ij}^{(m)} + F_{ij}^{(m-1)} - F_{ij}^{(m)} + \Phi_{ij}^{(m)} + J_{ij}^{(m)} - \epsilon_{ij}^{(m)} \tag{12}$$

where

$$P_{ij}^{(m)} = \int_{\kappa_{m-1}}^{\kappa_m} \mathcal{P}_{ij}(\boldsymbol{X},\kappa)d\kappa = -R_{ik}^{(m)}\frac{\partial \langle u_j \rangle}{\partial x_k} - R_{jk}^{(m)}\frac{\partial \langle u_i \rangle}{\partial x_k}, \tag{13}$$

$$F_{ij}^{(m)} = \mathcal{F}_{ij}^{(m)} - \varphi_{ij}(\boldsymbol{X},\kappa)\frac{\partial \kappa_m}{\partial t} \quad \textbf{and} \quad \mathcal{F}_{ij}^{(m)} = -\int_0^{\kappa_m} \mathcal{T}_{ij}(\boldsymbol{X},\kappa)d\kappa, \tag{14}$$

and

$$\Phi_{ij}^{(m)} = \int_{\kappa_{m-1}}^{\kappa_m} \Pi_{ij}(\boldsymbol{X},\kappa)d\kappa, \tag{15}$$

$$J_{ij}^{(m)} = \int_{\kappa_{m-1}}^{\kappa_m} \mathcal{J}_{ij}(\boldsymbol{X},\kappa)d\kappa, \tag{16}$$

$$\epsilon_{ij}^{(m)} = \int_{\kappa_{m-1}}^{\kappa_m} \mathcal{E}_{ij}(\boldsymbol{X},\kappa)d\kappa. \tag{17}$$

PARTIALLY INTEGRATED TRANSPORT MODELING (PITM)

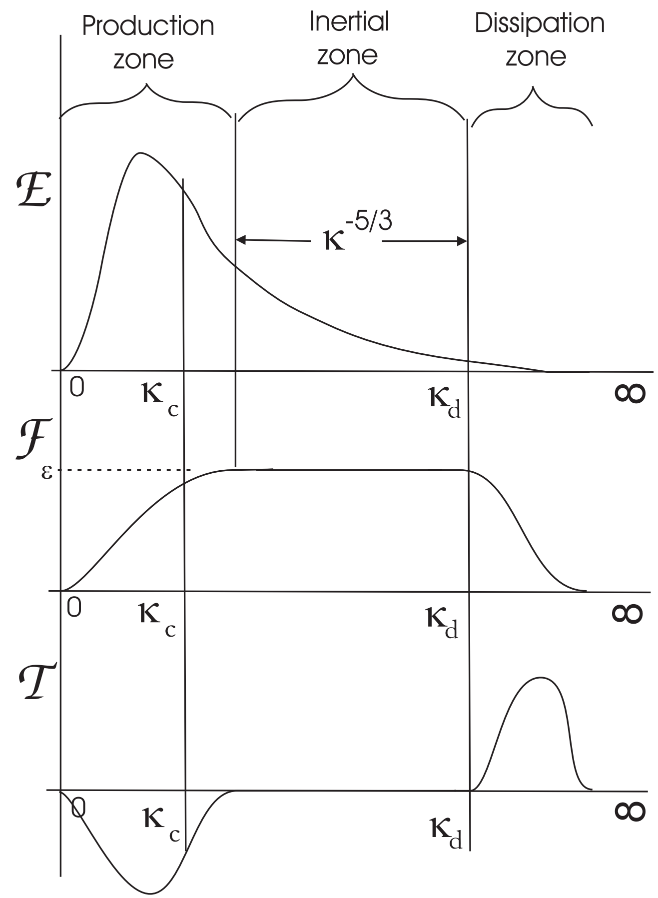

Figure 1: Turbulent processes in the spectral space

- **Wavenumber ranges such as** $[0, \kappa_c]$**,** $[\kappa_c, \kappa_d]$ **and** $[\kappa_d, \infty[$

PARTIALLY INTEGRATED TRANSPORT MODELING (PITM) METHOD

- **Transport equation for the partial turbulent energy**

$$\frac{Dk^{(m)}}{Dt} = P^{(m)} + F^{(m-1)} - F^{(m)} + J^{(m)} - \epsilon^{(m)} \quad \textbf{(18)}$$

- **Wavenumber ranges such as $[0, \kappa_c]$, $[\kappa_c, \kappa_d]$ and $[\kappa_d, \infty[$**

$$\frac{\partial(k - \langle k_{sfs} \rangle)}{\partial t} = P^{(1)} - F^{(1)}(\kappa_c) \quad \textbf{(19)}$$

$$\frac{\partial \langle k_{sfs} \rangle}{\partial t} = P^{(2)} - F^{(2)}(\kappa_d) + F^{(1)}(\kappa_c) \quad \textbf{(20)}$$

$$0 = F^{(2)}(\kappa_d) - \epsilon^{(3)} \quad \textbf{(21)}$$

where $\epsilon^{(3)} = \epsilon_{sfs} \approx \epsilon$. Equation (21) indicates that the dissipation rate ϵ can indeed be interpreted as a spectral flux.

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SPECTRAL SPLITTING

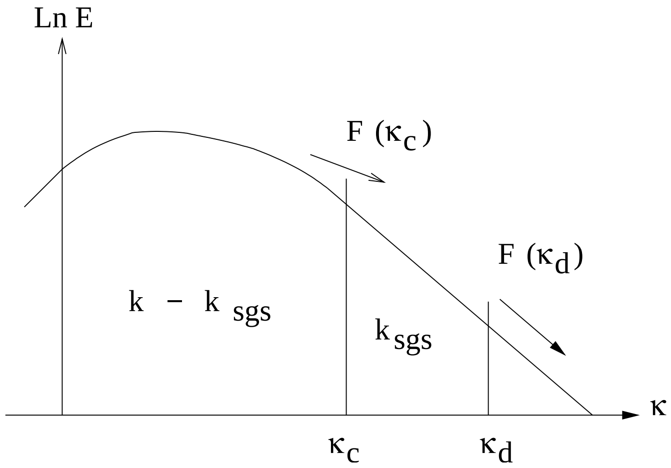

Figure 2: Turbulent processes in the spectral space

The splitting wavenumber κ_d is related to the cutoff wavenumber κ_c by the relation:

$$\kappa_d - \kappa_c = \zeta_c \frac{\epsilon}{\langle k_{sfs} \rangle^{3/2}} \quad \textbf{(22)}$$

In the case of full statistical modeling where $\kappa_c = 0$, equation (22) is reduced to the equation:

$$\kappa_d = \zeta_d \frac{\epsilon}{k^{3/2}} \quad \textbf{(23)}$$

where κ_d is located after the inertial range.

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FIRST FORMULATION OF THE DISSIPATION-RATE

The dissipation rate equation is then obtained by taking the derivative of equation

$$\kappa_d - \kappa_c = \zeta_c \frac{\epsilon}{\langle k_{sfs} \rangle^{3/2}}$$

One can easily obtain:

$$\frac{\partial \epsilon}{\partial t} = c_{sfs\epsilon_1} \frac{\epsilon}{\langle k_{sfs} \rangle} \left(P^{(2)} + F^{(1)}(\kappa_c) \right) - c_{sfs\epsilon_2} \frac{\epsilon^2}{\langle k_{sfs} \rangle} \quad (25)$$

where $c_{sfs\epsilon_1} = 3/2$ and

$$c_{sfs\epsilon_2} = \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{(\kappa_d - \kappa_c)\, E(\kappa_d)} \left[\left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d}{\epsilon} \right) - \frac{E(\kappa_d)}{E(\kappa_c)} \left(\frac{\mathcal{F}(\kappa_c) - F(\kappa_c)}{\epsilon} \right) \right] \quad (26)$$

Setting $E(\kappa_c) \gg E(\kappa_d)$ and $\kappa_d \gg \kappa_c$, then

$$c_{sfs\epsilon_2} \approx \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d}{\epsilon} \right) \quad (27)$$

SECOND FORMULATION OF THE DISSIPATION-RATE

In the case of full statistical modeling where $\kappa_c = 0$, Taking the time derivative of equation

$$\kappa_d = \zeta_d \frac{\epsilon}{k^{3/2}}$$

yields another formulation of the dissipation rate equation:

$$\frac{\partial \epsilon}{\partial t} = c_{\epsilon_1} \frac{\epsilon}{k} \left(P^{(1)} + P^{(2)} \right) - c_{\epsilon_2} \frac{\epsilon^2}{k} \quad \textbf{(29)}$$

where $c_{\epsilon_1} = 3/2$ and

$$c_{\epsilon_2} = \frac{3}{2} - \frac{k}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\epsilon} \right) \quad \textbf{(30)}$$

This is in fact the usual ϵ equation used in statistical closures.

FUNCTION $C_{sfs\epsilon_2}$

- **Analytical expression of the function $c_{sfs\epsilon_2}$**

$$c_{sfs\epsilon_2} \approx \frac{3}{2} - \frac{\langle k_{sfs} \rangle}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\epsilon} \right) \tag{31}$$

$$c_{\epsilon_2} = \frac{3}{2} - \frac{k}{\kappa_d E(\kappa_d)} \left(\frac{\mathcal{F}(\kappa_d) - F(\kappa_d)}{\epsilon} \right) \tag{32}$$

$$c_{sfs\epsilon_2} = c_{\epsilon_1} + \frac{\langle k_{sfs} \rangle}{k} \left(c_{\epsilon_2} - c_{\epsilon_1} \right) \tag{33}$$

- **Equations (31) and (32) show that the coefficients $c_{sfs\epsilon_2}$ and c_{ϵ_2} are functions of the spectrum shape**

- **Note that the fluxes $\mathcal{F}$ and F can be analytically computed in some particular situations**

- **Computation of ratio $\langle k_{sfs} \rangle / k$**

$$E(\kappa) = \frac{\frac{2}{3}\beta_\eta L_e^3 \, k \, \kappa^2}{\left[1 + \beta_\eta (\kappa \, L_e)^3)\right]^{11/9}} \tag{34}$$

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PITM METHOD

- **Instantaneous transport equations and practical formulations**

$$\frac{Dk_{sfs}}{Dt} = P_{sfs} - \epsilon_{sfs} + J_{sfs} \qquad \textbf{(35)}$$

$$\frac{D(\tau_{ij})_{sfs}}{Dt} = (P_{ij})_{sfs} - (\epsilon_{ij})_{sfs} + (\Phi_{ij})_{sfs} + (J_{ij})_{sfs} \qquad \textbf{(36)}$$

$$\frac{D\epsilon_{sfs}}{Dt} = c_{\epsilon_1} \frac{\epsilon_{sfs}}{k_{sfs}} \frac{(P_{mm})_{sfs}}{2} - c_{sfs\epsilon_2}(\eta_c) \frac{\epsilon^2_{sfs}}{k_{sfs}} + (J_\epsilon)_{sfs} \qquad \textbf{(37)}$$

  - **"Exact " coefficient c_{ϵ_2} where $\eta_c = (k^{3/2}\kappa_c)/(\epsilon_{sfs} + \epsilon^<)$**

$$c_{sfs\epsilon_2}(\eta_c) = c_{\epsilon_1} + \frac{c_{\epsilon_2} - c_{\epsilon_1}}{[1 + \beta_\eta\, \eta_c^3]^{2/9}} \qquad \textbf{(38)}$$

  - **Dynamic procedure (Friess et al., 2010) with $r_{Eq} = (\langle k_{sfs}\rangle / k)_{Eq}$**

$$\delta c_{sfs\epsilon_2} = \chi\, c_{sfs\epsilon_2} \left(1 - \frac{r_{CFD}}{r_{Eq}}\right) \qquad \textbf{(39)}$$

NUMERICAL METHOD

- **Three-dimensional compressible code for solving (5+7) transport equations (Chaouat, 2010)**
  - ρ, u_1, u_2, u_3, E
  - $(\tau_{11})_{sfs}, (\tau_{12})_{sfs}, (\tau_{13})_{sfs}, (\tau_{22})_{sfs}, (\tau_{23})_{sfs}, (\tau_{33})_{sfs}, \epsilon_{sfs}$
- **Finite volumes technique: fluxes conservative method**

$$\frac{\partial \boldsymbol{U}}{\partial t} = -\frac{1}{v(\Omega)} \sum_{\sigma} (\boldsymbol{F} - \boldsymbol{F}_{\mathrm{V}}) A_\sigma + \boldsymbol{S} \qquad (40)$$

where $\boldsymbol{F}$ and $\boldsymbol{F}_{\mathrm{V}}$ represent respectively the convective and viscous fluxes through the surfaces A_σ around the control volume $v(\Omega)$, $\boldsymbol{n}$ is the unit vector normal to the surface A_σ and $\boldsymbol{S}$ is the source term.

- **Fourth order Runge-Kutta scheme in time discretization**
- **Implicit scheme in time for the treatment of the turbulent equations**
- **Second and fourth order centered schemes in space discretization (MUSCL scheme)**
- **CPU time : the subfilter-scale stress model (7 transport equations) only requires 25 % more time than the viscosity model (2 transport equations)**

INJECTION INDUCED FLOWS

- No slip condition for the upper surface
- Injection condition for the lower surface
- Numerical simulations performed on different grids
  - **PITM1 (**$400 \times 32 \times 80) \approx 1.0\,10^6$
  - **PITM2 (**$400 \times 44 \times 80) \approx 1.4\,10^6$
- Comparison with highly resolved LES (Apte and Yang, 2003) performed on a refined grid $8.4\,10^6$ **(**$N_{LES}/N_{PITM2} = 6$**)**

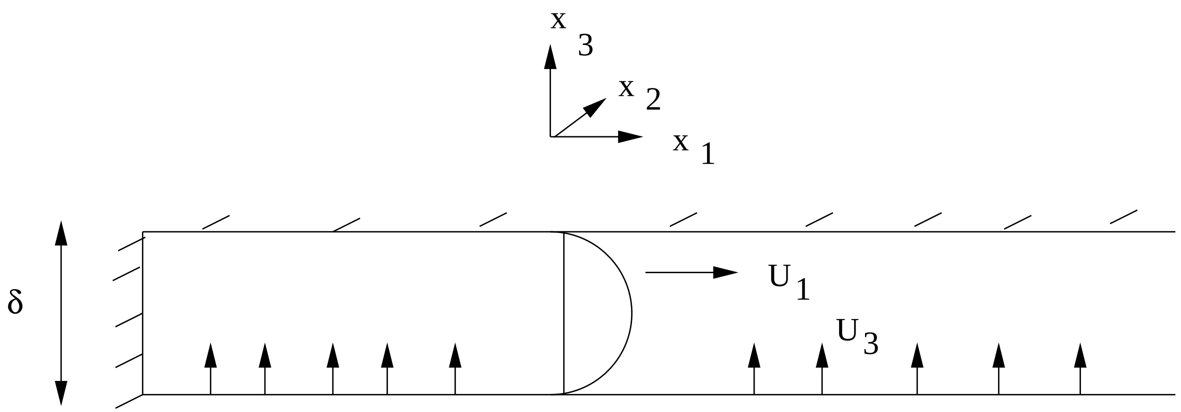

Figure 3: Schematic of channel flow with fluid injection.

INJECTION INDUCED FLOWS

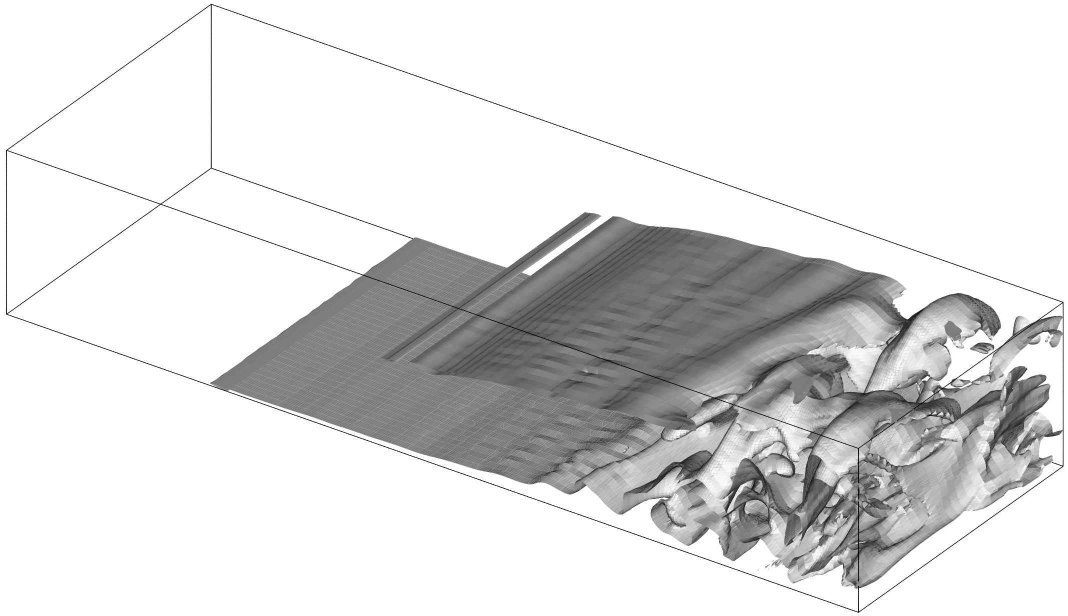

Figure 4: Isosurfaces of instantaneous filtered vorticity $\bar{\omega}_i = \epsilon_{ijk}\partial\bar{u}_k/\partial x_j$ in the spanwise direction (i=2) $|\bar{\omega}_2| = 3000$ (1/s). (Chaouat and Schiestel, 2007)

- **Transitional laminar-turbulent flow**
- **The three-dimensional structures are squeezed upward in the normal direction to the wall**

INJECTION INDUCED FLOWS

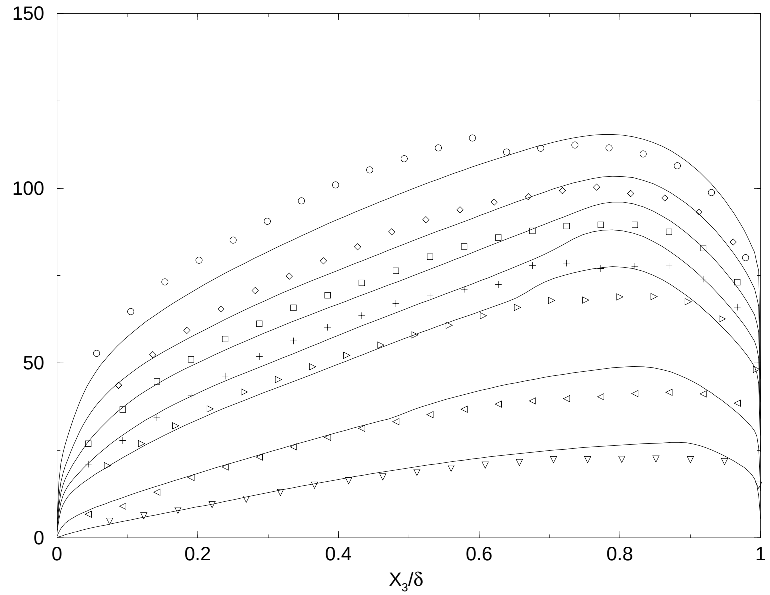

Figure 5: Mean velocity profiles in different cross sections. $x_1 = 12$ cm: $\bigtriangledown$; 22 cm: $\triangleleft$; 35 cm: $\triangleright$; 40 cm : $+$; 45 cm: $\square$; 50 cm: $\Diamond$; 57 cm: $\circ$. —–: PITM; Symbols: experimental data (Avalon, 2000)

INJECTION INDUCED FLOWS

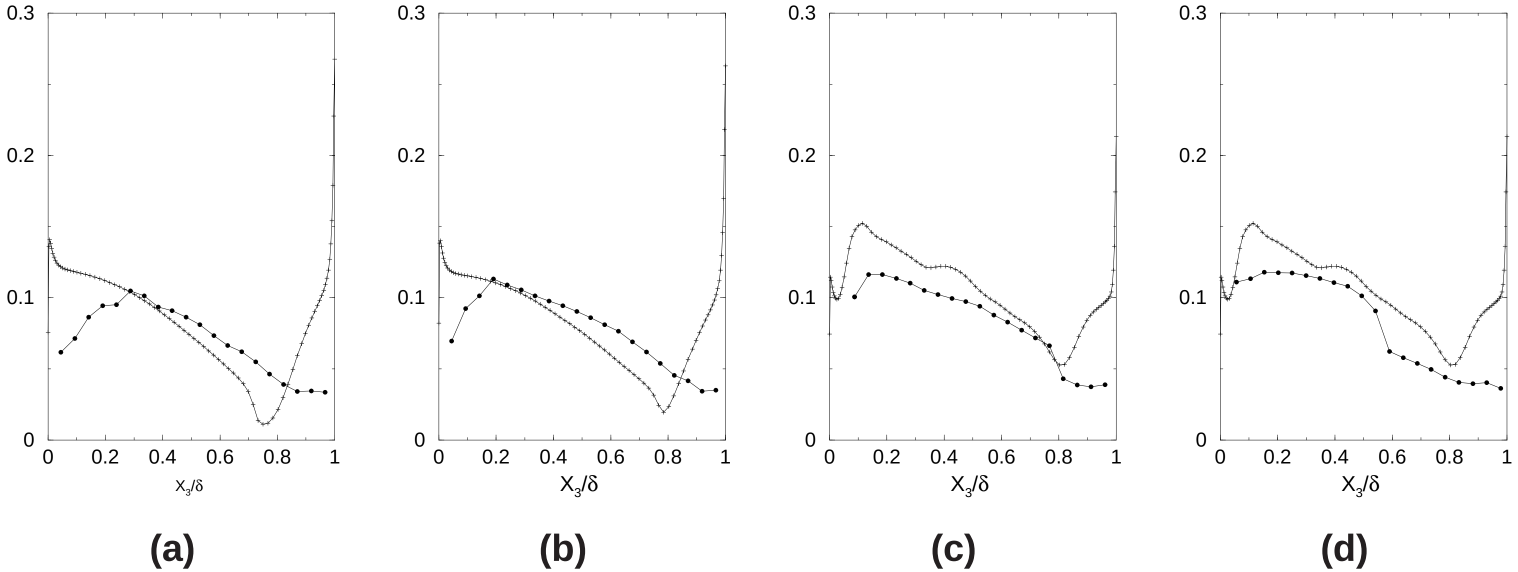

Figure 6: Streamwise turbulent stresses $< (\tau_{11})^{\frac{1}{2}} > / u_m$ in different cross sections (a) $x_1 =$ 40 cm; (b) 45 cm; (c) 50 cm; (d) 57 cm. + : PITM ; ●: experimental data (Avalon, 2000)

SIMULATION OF CHANNEL FLOWS OVER PERIODIC HILLS

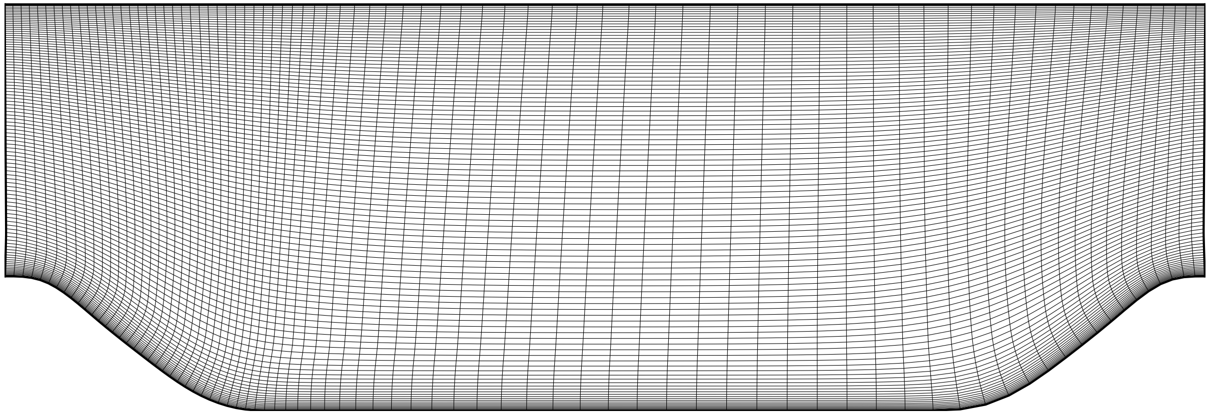

Figure 7: Cross-section of the curvilinear grid (80 $\times$ 100) of the contracted channel

- Numerical simulation performed on a coarse and medium grids $24\,10^5; 10^6$, (Chaouat, 2010)
- Comparison with highly resolved LES performed on refined grids $5\,10^6$ (Fröhlich et al., 2005) and $13\,10^6$ (Breuer et al., 2009) ($N_{LES}/N_{PITM1} \approx 54.2$)
- Turbulence mechanisms associated with separation, recirculation, reattachment and acceleration that are difficult to reproduce using RANS/RSM models

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SIMULATION OF CHANNEL FLOWS OVER PERIODIC HILLS

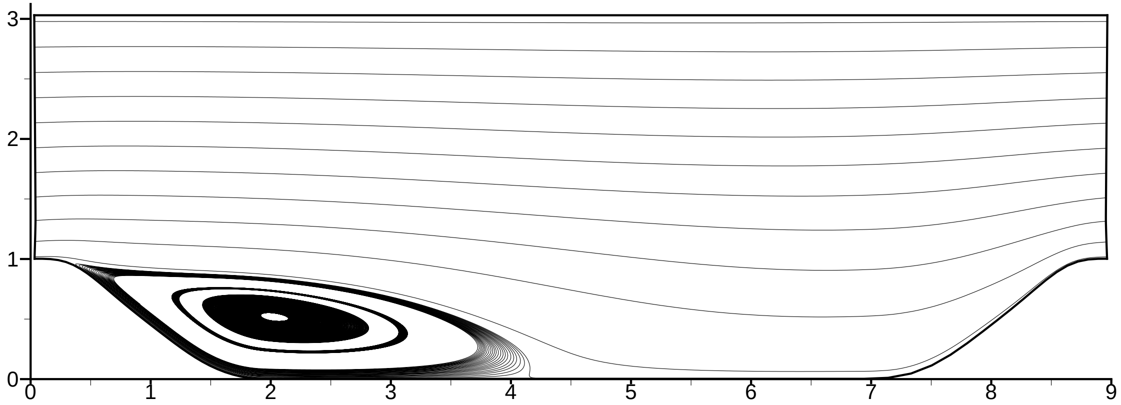
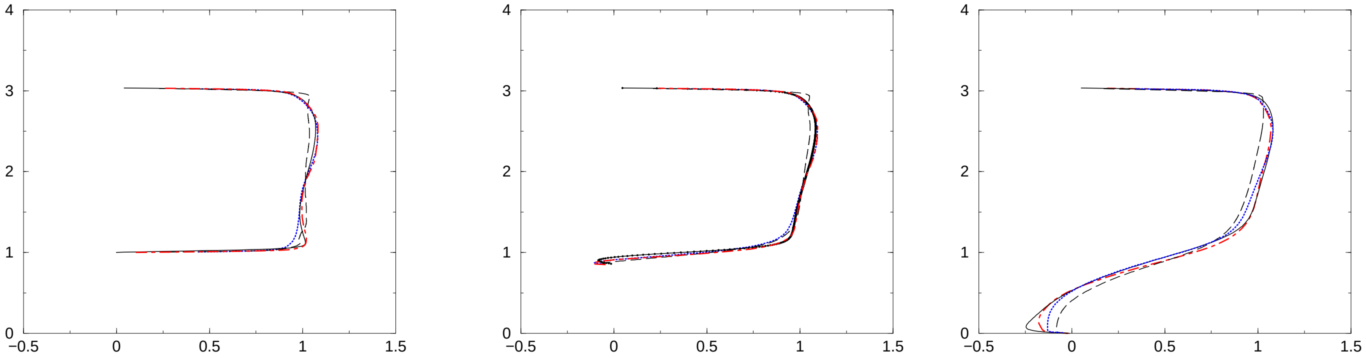


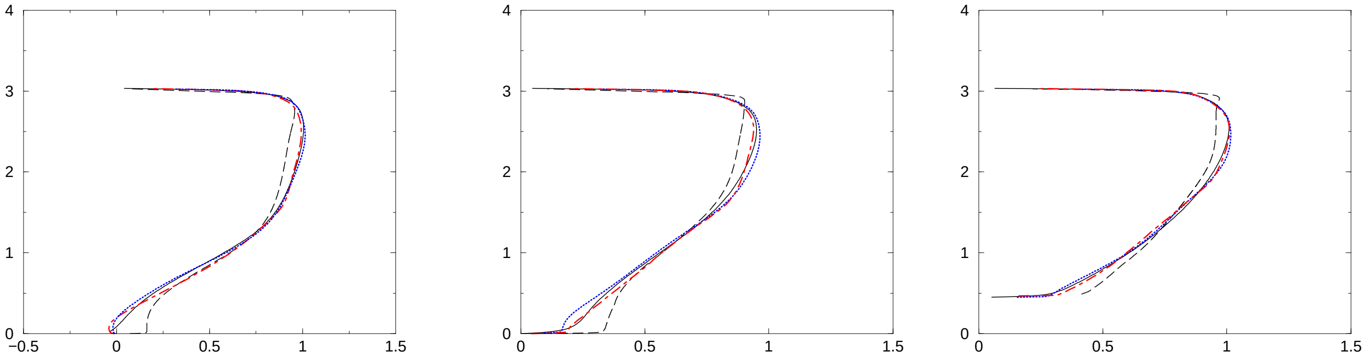
Figure 8: Streamlines of the averaged flowfield

- **The flow statistically separates at $x_1/h \approx 0.23$ downstream the hill and reattaches at $x_1/h \approx 4.3$ in good agreement with the highly resolved LES data (Fröhlich et al., 2005) and (Breuer et al., 2009)**

SIMULATION OF CHANNEL FLOWS OVER PERIODIC HILLS



(a) $\langle u_1 \rangle / U_b$. $x_1/h = 0.05$ **(b)** $x_1/h = 0.5$ **(c)** $x_1/h = 2$



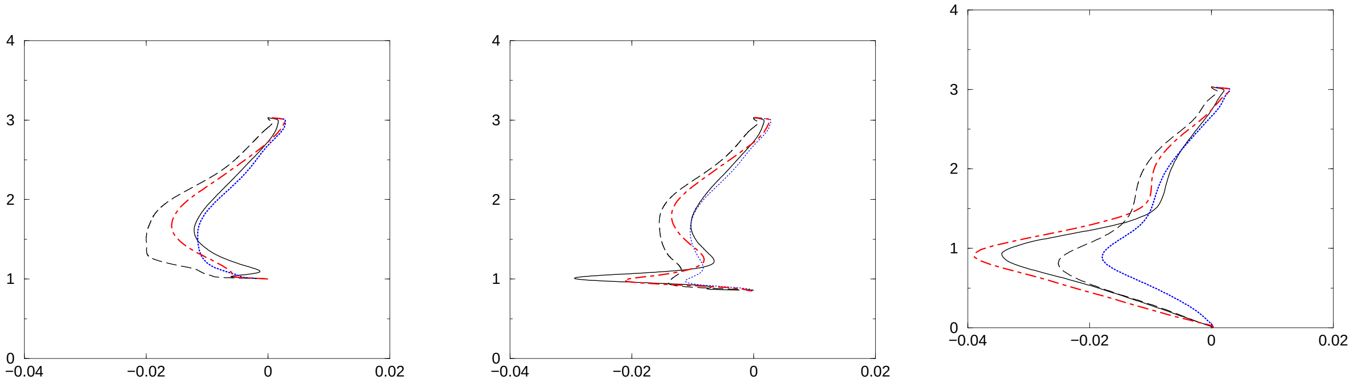
(d) $x_1/h = 4$ **(e)** $x_1/h = 6$ **(f)** $x_1/h = 8$

Figure 9. Streamwise velocity $\langle u_1 \rangle / U_b$ at different locations ($x_1/h = 0.05$, 0.5, 2, 4, 6, 8.

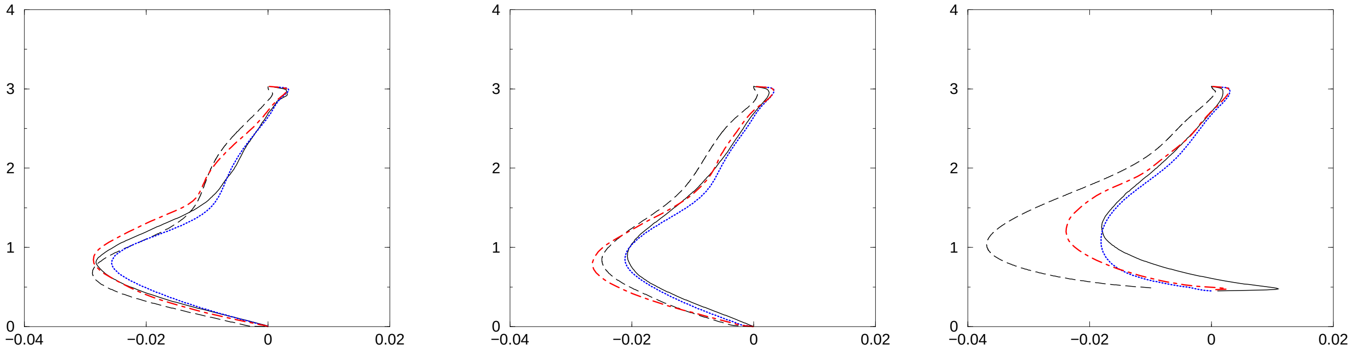
LES (Breuer *et al.*, 2009) —; PITM1 ($80 \times 30 \times 100$ $\cdots$;

PITM2 ($160 \times 60 \times 100$) -.-.; RSM - -

SIMULATION OF CHANNEL FLOWS OVER PERIODIC HILLS



(a) τ_{13}/U_b^2**.** $x_1/h = 0.05$ **(b)** $x_1/h = 0.5$ **(c)** $x_1/h = 2$



(d) $x_1/h = 4$ **(e)** $x_1/h = 6$ **(f)** $x_1/h = 8$

Figure 10. Turbulent shear stress τ_{13}/U_b^2 at different locations ($x_1/h = 0.05$, 0.5, 2, 4, 6, 8.

LES (Breuer *et al.*, 2009) —; PITM1 ($80 \times 30 \times 100$) $\cdots$;

PITM2 ($160 \times 60 \times 100$) -.-.; RSM - -

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION

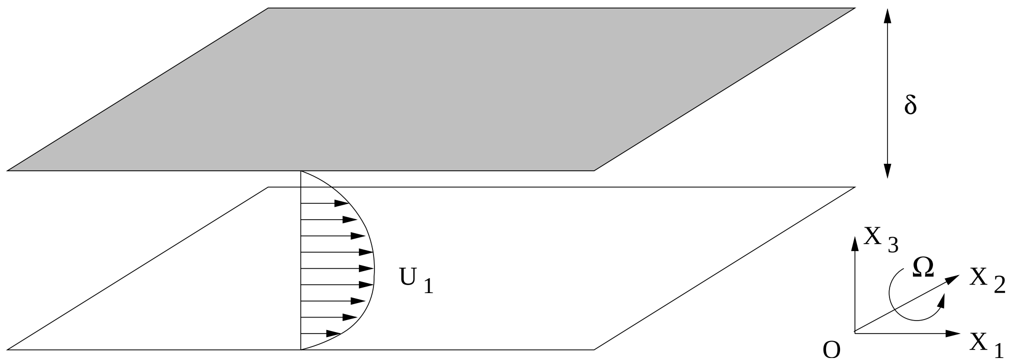
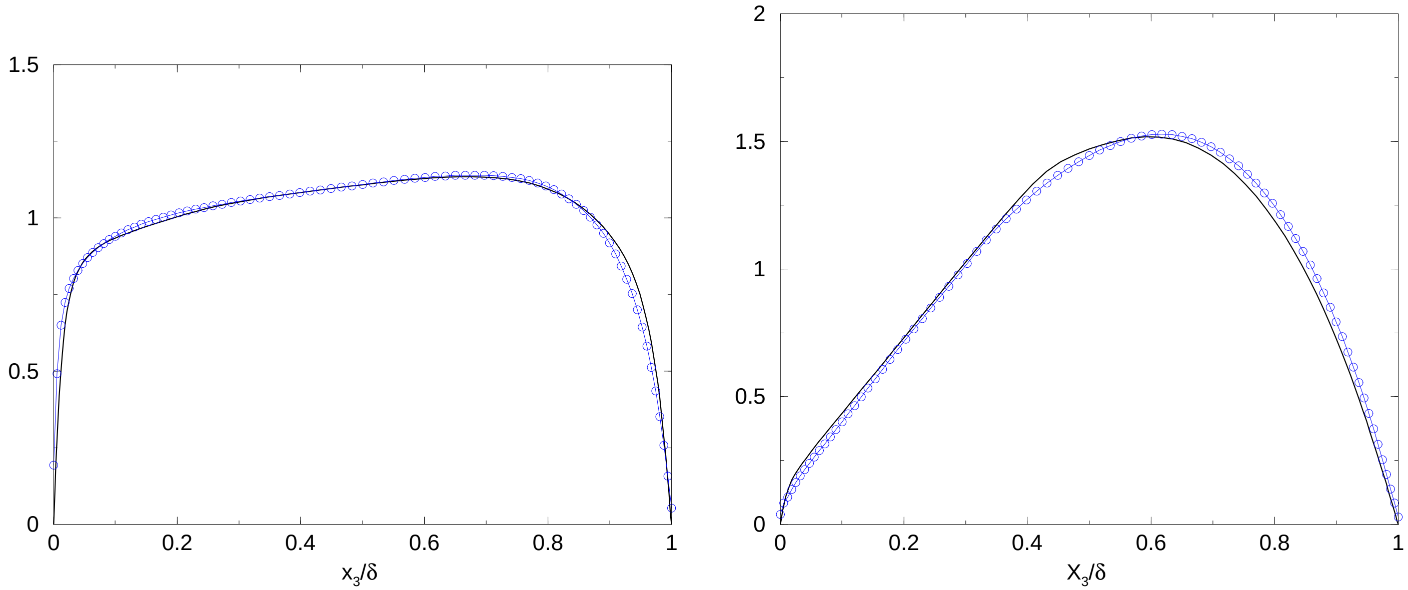


Figure 11: Schematic of fully-developed turbulent channel flow in a rotating frame.

- **Coarse grids are deliberately chosen to highlight the ability of the PITM method to simulate large scales of the flow**
  - **PITM1** $(24 \times 48 \times 64) \approx 7\,10^4$
  - **PITM2** $(84 \times 64 \times 64) \approx 3.4\,10^5$
- **Comparison with highly resolved LES (Lamballais et al., 1998) performed on a refined grid** $8\,10^5$ **(**$N_{LES}/N_{PITM1} \approx 11.4$**)**

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION



(a) **(b)**

Figure 12: Mean velocity profile $\langle u_1 \rangle / u_m$ in global coordinate. PITM1 $(24 \times 48 \times 64)$: ○; Highly resolved LES (Lamballais 1998): — . $R_m = 14000$, (a) $Ro_m = 0.17$. (b) $Ro = 1.5$

- **As the rotation rate increases:**
  - **breaking of the symmetry (mean velocity and turbulent stresses)**
  - **destabilization effects on the anticyclonic flow region**
  - **relaminarization effects on the cyclonic flow region**

CHANNEL FLOWS SUBJECTED TO A SPANWISE ROTATION

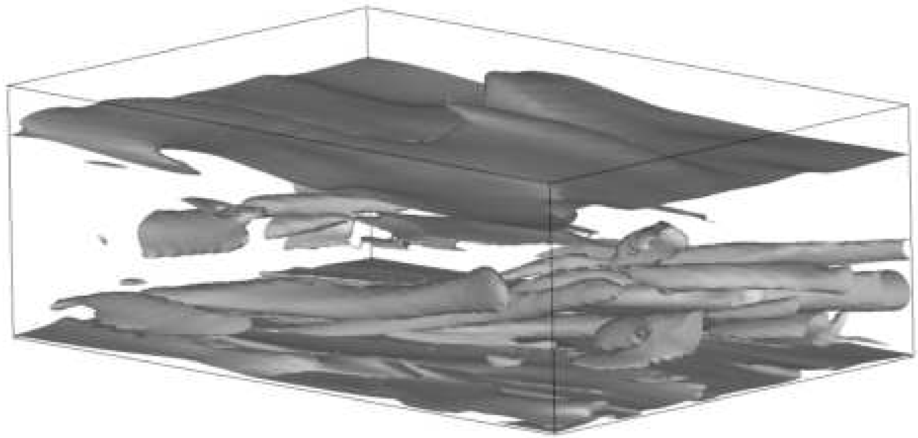


Figure 13: Isosurfaces of vorticity modulus $\omega = 3u_m/\delta = 12.\,10^5$. $R_m = 14000$**,** $R_o = 1.50$**.**
PITM2 $(84 \times 64 \times 64)$

- **As the rotation rate increases:**
  - **the structures become more and more organized in the anticyclonic wall region**
  - **the structures are less inclined with respect to the wall** $(\alpha < 45°)$

ONERA

CONCLUSION

- Partial integrated transport modeling (PITM) method
  - Mathematical physics formalism developed in the spectral space
  - Continuous hybrid non-zonal RANS/LES simulations performed on coarse grids
  - Drastic reductions of the computational cost by coarsening the meshes
- PITM is a method and not a model !
  - PITM can be applied to each RANS model to derive its corresponding subfilter model
- Engineering applications
  - Simulations of turbulent flows that present a complex physics
  - Unsteady flows with non-standard spectral distribution (some departure from the Kolmogorov spectrum)

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MATHEMATICAL PHYSICS FORMALISM IN SPECTRAL SPACE

- **Dynamic equation for the double velocity correlation (Hinze, 1975)**

$$
\begin{aligned}
&\frac{\partial R_{ij}(\boldsymbol{X},\boldsymbol{\xi})}{\partial t} + \frac{1}{2}\big(\langle u_{kA}\rangle + \langle u_{kB}\rangle\big)\frac{\partial R_{ij}(\boldsymbol{X},\boldsymbol{\xi})}{\partial X_k} = \\
&-R_{jk}(\boldsymbol{X},\boldsymbol{\xi})\left(\frac{\partial\langle u_i\rangle}{\partial x_k}\right)_A - R_{ik}(\boldsymbol{X},\boldsymbol{\xi})\left(\frac{\partial\langle u_j\rangle}{\partial x_k}\right)_B \\
&-\big(\langle u_{kB}\rangle - \langle u_{kA}\rangle\big)\frac{\partial R_{ij}(\boldsymbol{X},\boldsymbol{\xi})}{\partial \xi_k} \\
&-\frac{1}{2}\frac{\partial}{\partial X_k}\Big(\Big\langle u'_{iA}u'_{kB}u'_{jB}\Big\rangle + \langle u'_{iA}u'_{kA}u'_{jB}\rangle\Big)(\boldsymbol{X},\boldsymbol{\xi}) \\
&-\frac{\partial}{\partial \xi_k}\Big(\langle u'_{iA}u'_{kB}u'_{jB}\rangle - \langle u'_{iA}u'_{kA}u'_{jB}\rangle\Big)(\boldsymbol{X},\boldsymbol{\xi}) \\
&-\frac{1}{2\rho}\left(\frac{\partial}{\partial X_i}\langle p'_A u'_{jB}\rangle + \frac{\partial}{\partial X_j}\langle p'_B u'_{iA}\rangle\right)(\boldsymbol{X},\boldsymbol{\xi}) \\
&+\frac{1}{\rho}\left(\frac{\partial}{\partial \xi_i}\langle p'_A u'_{jB}\rangle - \frac{\partial}{\partial \xi_j}\langle p'_B u'_{iA}\rangle\right)(\boldsymbol{X},\boldsymbol{\xi}) \\
&+\frac{\nu}{2}\frac{\partial^2 R_{ij}}{\partial X_l \partial X_l}(\boldsymbol{X},\boldsymbol{\xi}) + 2\nu\frac{\partial^2 R_{ij}}{\partial \xi_l \partial \xi_l}(\boldsymbol{X},\boldsymbol{\xi})
\end{aligned}
\qquad \textbf{(41)}
$$